SAMPLE 10.5 A SDOF spring-mass model from given data: The following table is obtained for successive peaks of displacement from the simulation of free vibration of a mechanical system. Make a single degree of freedom mass-spring-dashpot model of the system choosing appropriate values for mass, spring stiffness, and damping rate.
Data:

peak no. n	0	1	2	3	4	5	6
time (s)	0.0000	0.6279	1.2558	1.8837	2.5116	3.1395	3.7674
peak x (m)	0.5006	0.4697	0.4411	0.4143	0.3892	0.3659	0.3443

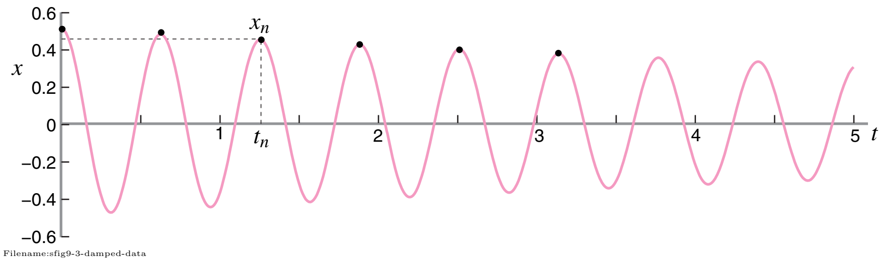

Figure 10.12: Oscillation data from the simulation of a mechanical system

Solution Since the data provided is for successive peak displacements, the time between any two successive peaks represents the period of oscillations. It is also clear that the system is underdamped because the successive peaks are decreasing. We can use the logarithmic decrement method to determine the damping in the system.

First, we find the time period T_D from which we can determine the damped circular frequency λ_D. From the given data we find that

$$t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \cdots = 0.6279\,\text{s}$$

Therefore,

$$\begin{aligned} T_D &= 0.6279\,\text{s}. \\ \Rightarrow \quad \lambda_D &= \frac{2\pi}{T_D} = 10\,\text{rad/s}. \end{aligned} \tag{10.21}$$

Now we make a table for the logarithmic decrement of the peak displacements:

peak disp. x_n (m)	0.5006	0.4697	0.4411	0.4143	0.3892	0.3659	0.3443
$\frac{x_n}{x_{n+1}}$	1.0658	1.0648	1.0647	1.0645	1,0637	1.0627	
$\ln\left(\frac{x_n}{x_{n+1}}\right)$	0.0637	0.0628	0.0627	0.0624	0.0618	0.0608	

③ Theoretically, all of these values should be the same, but it is rarely the case in practice. When x_n's are measured from an experimental setup, the values of D may vary even more.

Thus, we get several values of the logarithmic decrement $D = \ln\left(\frac{x_n}{x_{n+1}}\right)$ ③

We take the average value of D:

$$D = \bar{D} = 0.0624. \tag{10.22}$$

Let the equivalent single degree of freedom model have mass m, spring stiffness k, and damping rate c. Then

$$\lambda_D = \lambda\sqrt{1-\xi^2} \approx \lambda = \sqrt{\frac{k}{m}}.$$

Thus, from Eqn (10.21),

$$\frac{k}{m} = \lambda^2 = 100(\,\text{rad/s})^2, \tag{10.23}$$

and, since $D = \frac{cT_D}{2m}$, from Eqn (10.22) we get

$$\begin{aligned} c &= \frac{2mD}{T_D} \\ &= \frac{2m(0.0624)}{0.6279\,\text{s}} \\ &= (0.1988\frac{1}{\text{s}})m. \end{aligned} \tag{10.24}$$

Equations (10.23) and (10.24) have three unknowns: k, m, and c. We cannot determine all three uniquely from the given information. So, let us pick an arbitrary mass $m = 5\,\text{kg}$. Then

$$\begin{aligned} k &= (100\frac{1}{\text{s}^2})\cdot(5\,\text{kg}) \\ &= 500\,\text{N/m}, \end{aligned}$$

and

$$\begin{aligned} c &= (0.1988\frac{1}{\text{s}})\cdot(5\,\text{kg}) \\ &= 0.99\,\text{N}\cdot\text{s/m}. \end{aligned}$$

$$\begin{aligned} m &= 5\,\text{kg}, \\ k &= 500\,\text{N/m}, \\ c &= 0.99\,\text{N}\cdot\text{s/m}. \end{aligned}$$

Of course, we could choose many other sets of values for m, k, and c which would match the given response. In practice, there is usually a little more information available about the system, such as the mass of the system. In that case, we can determine k and c uniquely from the given response.