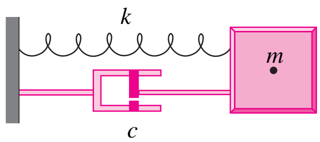

Figure 10.19:

SAMPLE 10.6 The mass-spring-dashpot system shown in the figure consists of a mass $m = 2\,\text{kg}$, a spring with stiffness $k = 3200\,\text{N/m}$ and a dashpot with damping coefficient $c = 10\,\text{kg/s}$.

1. Is the system underdamped, critically damped or overdamped?
2. Find the damped natural frequency of the system.
3. What is the resonant frequency of the system.

Solution

1. The question about underdamped, critically damped, or overdamped can be answered conveniently by computing the damping ratio ξ. For an underdamped system, $\xi < 1$, for a crtically damped system, $\xi = 1$, and for an overdamped system $\xi > 1$. So, let us compute ξ. We know that
$$\xi = \frac{c}{c_c} = \frac{c}{2\sqrt{km}}.$$
Thus, for the given system,
$$\xi = \frac{10\,\text{kg/s}}{2\sqrt{3200\,\text{N/m}\cdot 2\,\text{kg}}} = \frac{10\,\text{kg/s}}{160\,\text{kg/s}} = 0.062.$$
Since $\xi < 1$, the system is underdamped.

Underdamped ($\xi = 0.062$)

2. The damped natural frequency, λ_d, is given by
$$\lambda_d = \lambda_n\sqrt{1-\xi^2}$$
where $\lambda_n = \sqrt{k/m}$ is the natural frequency of the system. Substituting the known values, we get
$$\lambda_d = \sqrt{\frac{3200\,\text{N/m}}{2\,\text{kg}}}\sqrt{1-(0.062)^2} = 39.92\,\text{rad/s}$$
which is almost the same as the natural frequency $\lambda_n = 40\,\text{rad/s}$.

$\lambda_d = 39.92\,\text{rad/s}$

3. The resonant frequency of the system, λ_r, is given by
$$\lambda_r = \lambda_n\sqrt{1-2\xi^2}.$$
Substituting the known values of λ_n and ξ, we get
$$\lambda_r = 39.85\,\text{rad/s}$$
which is the smallest among the three characteristic frequencies of the system — natural frequency, damped natural frequency, and the resonant frequency. For small values of ξ, however, the three frequencies are practically indistiguishable as is the case here.

$\lambda_r = 39.85\,\text{rad/s}$

Introduction to Statics and Dynamics, © Andy Ruina and Rudra Pratap 1994-2013.