SAMPLE 10.8 A single degree of freedom damped oscillator has unknown mass, spring stiffness and damping coefficient. In order to find these quantities, the oscillator is subjected to a constant force $F_0 = 100\,\text{N}$ and its transient response is recorded. The response is shown in fig. 10.24. The two peaks marked in the response plot correspond to (t, x) = (0.2107 s, 0.01345 m) and (0.3525 s, 0.0117 m) respectively. Find the system parameters m, k, and c.

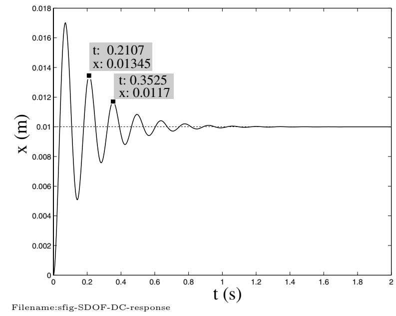

Figure 10.23:

Solution Let the mass, stiffness, and damping coefficient of the system be m, k, and c, respectively. Then the equation of motion of the system, subjected to a constant force F_0 is,

$$m\ddot{x} + c\dot{x} + kx = F_0$$

where $x(t)$ is the displacement at some instant t. From the solution of this equation, we know that the steady state solution (after the transient oscillations die) is merely a shift in the static equilibrium position, given by F_0/k. From the given response, we see that

$$\frac{F_0}{k} = 0.01\,\text{m} \quad \Rightarrow \quad k = \frac{F_0}{0.01\,\text{m}} = \frac{100\,\text{N}}{0.01\,\text{m}} = 10\,\text{kN/m}.$$

Thus we have found one of the parameters, k. Now we need to find m and c.

Since two successive peaks are given in the transient response, we can use the logrithmic decrement to determine the damping ratio ξ from the relationship

$$\xi = \frac{1}{2\pi}\ln\left(\frac{x_n}{x_{n+1}}\right).$$

From the given data, $x_n = 0.01345\,\text{m}$ and $x_{n+1} = 0.0117\,\text{m}$. Therefore, (5)

$$\xi = \frac{1}{2\pi}\ln\left(\frac{0.01345\,\text{m}}{0.0117\,\text{m}}\right) = 0.022.$$

Since $\xi = c/c_c = c/(2\sqrt{km})$, we have

$$c = 2\xi\sqrt{km} = 0.044\sqrt{km} \tag{10.43}$$

This is just one equation in two unknowns, m and c (we already know k). So, we need another equation. From the peak to peak distance (in time), we can find the damped time period. That is $T_d = T_2 - T_1 = 0.3525\,\text{s} - 0.2107\,\text{s} = 0.1418\,\text{s}$. But, $T_d = 2\pi/\lambda_d$, and $\lambda_d = \lambda_n\sqrt{1-\xi^2}$. Therefore,

$$\begin{aligned}
\lambda_n^2 \equiv \frac{k}{m} &= \frac{\lambda_d^2}{1-\xi^2} = \frac{4\pi^2}{T_d^2(1-\xi^2)} \\
\Rightarrow \quad m &= \frac{kT_d^2(1-\xi^2)}{4\pi^2} \\
&= \frac{10000\,\text{N/m}\cdot(0.1418\,\text{s})^2(1-0.022^2)}{4\pi^2} \\
&= 5.09\,\text{kg}.
\end{aligned}$$

Now substituting the value of m and k in eqn. (10.43), we get

$$c = 0.044\sqrt{10000\,\text{N/m}\cdot 5.09\,\text{kg}} = 9.92\,\text{kg/s}.$$

$$m = 5.09\,\text{kg}, \quad k = 10\,\text{kN/m}, \text{ and } c = 9.92\,\text{kg/s}.$$

(5) The calculation here is illustrative. In practice, you should not use just two data points for computing damping from the logarithmic decrement method. In general, we use several data points and determine the damping from taking an average of the decrements. See Sample 10.5 on page 512.