SAMPLE 10.9 Damping and forced response: When a single-degree-of-freedom damped oscillator (mass-spring-dashpot system) is subjected to a periodic forcing $F(t) = F_0 \sin(pt)$, then the response of the system is given by

$$x(t) = X\cos(pt - \phi)$$

where $X = \frac{F_0/k}{\sqrt{(2\xi r)^2 + (1-r^2)^2}}$, $\phi = \tan^{-1}\frac{2\xi r}{1-r^2}$, $r = \frac{p}{\lambda}$, $\lambda = \sqrt{k/m}$ and ξ is the damping ratio.

1. For $r \ll 1$, *i.e.,* the forcing frequency p much smaller than the natural frequency λ, how does the damping ratio ξ affect the response amplitude X and the phase ϕ?
2. For $r \gg 1$, *i.e.,* the forcing frequency p much larger than the natural frequency λ, how does the damping ratio ξ affect the response amplitude X and the phase ϕ?

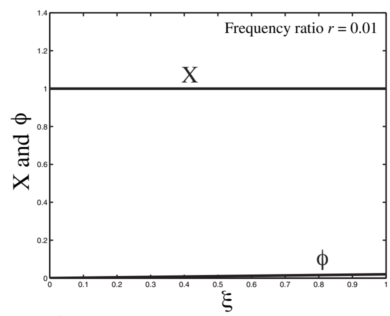

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Figure 10.24:

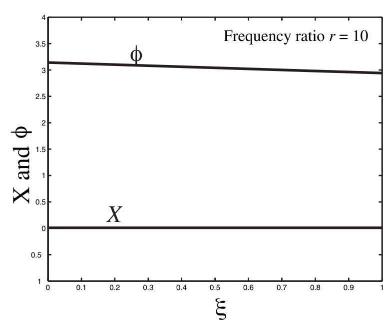

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Figure 10.25:

Solution

1. If the frequency ratio $r \ll 1$, then r^2 will be even smaller; so we can ignore r^2 terms with respect to 1 in the expressions for X and ϕ. Thus, for $r \ll 1$,

$$X = \frac{F_0/k}{\sqrt{(2\xi r)^2 + (1-r^2)^2}} \approx \frac{F_0/k}{1} = \frac{F_0}{k}$$
$$\phi = \tan^{-1}(2\xi r) \approx \tan^{-1} 0 = 0$$

that is, the response amplitude does not vary with the damping ratio ξ, and the phase also remains constant at zero. As an example, we use the full expressions for X and ϕ for plotting them against ξ for $r = 0.01$ in fig. 10.25

For $r \ll 1$, $X \approx F_0/k$, and $\phi \approx 0$

2. If $r \gg 1$, then the denominator in the expression for X, $4\xi^2 r^2 + (1-r^2)^2 \approx r^4$ (because we can ignore all other terms with respect to r^4. Similarly, we can ignore 1 with respect to r^2 in the expression for ϕ. Thus, for $r \gg 1$,

$$X = \frac{F_0/k}{\sqrt{(2\xi r)^2 + (1-r^2)^2}} \approx \frac{F_0/k}{r^2} = 0$$
$$\phi = \tan^{-1}\frac{2\xi r}{-r^2} \approx \tan^{-1}\frac{2\xi}{-r} \approx \tan^{-1}(-0) = \pi.$$

Once again, we see that the response amplitude and phase do not vary with ξ. This is also evident from fig. 10.26 where we plot X and ϕ using their full expressions for $r = 10$. The slight variation in ϕ around π goes away as we take higher values of r.

For $r \gg 1$, $X \approx 0$, and $\phi \approx \pi$

Thus, we see that the damping in a system does not affect the response of the system much if the forcing frequency is far away from the natural frequency.