SAMPLE 10.12 A two mass vibratory MEMS gyroscope: A vibratory MEMS (microelectromechanical system) gyroscope employs two big plates as inertial masses, suspended by thin beams or 'springs' as shown in the figure. The two masses are made to vibrate (by electrical actuation) out of phase in the x-direction. Any rotation about the y-direction causes the masses to vibrate out of plane due to 'Coriolis acceleration' (you will learn about that in later chapters). We will restrict our attention to the planar motion of the gyroscope. A two degree of freedom spring-mass model is shown in the figure where $m = 34.5 \times 10^{-9}$ kg, $k_1 = 25$ N/m, and $k_2 = 3$ N/m.

1. Write the equations of motion for the two masses.
2. For the out of phase motion of the two masses, assume that $x_1(t) = -x_2(t) = x_0 \sin \lambda_n t$. Determine the natural frequency λ_n corresponding to this mode of vibration.

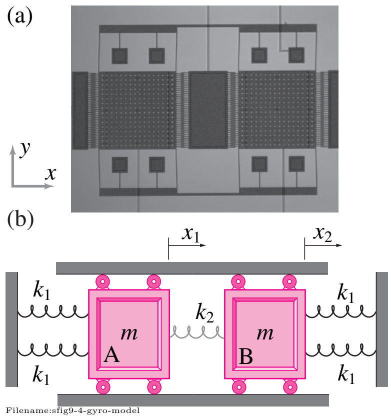

Figure 10.31: A MEMS vibratory gyroscope: (a) a micrograph of the two-mass structure. Each inertial mass (the plates with holes) is approximately 1 mm × 1 mm × 15μ m. The beams that hang the two masses act as springs. The comb drives on the left and right side of the two masses are used to drive the two masses to oscillate in the x-direction at their resonant frequencies. (b) a two degree of freedom spring-mass model of the driven gyroscope structure (without any damping).

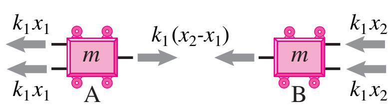

Figure 10.32: Partial free body diagram of the two masses. Only relevant forces (in the x-direction) are shown in the diagram.

Solution

1. The free body diagram of each mass is shown in shown in fig. 10.33. Assuming both x_1 and x_2 to be positive to in the x-direction, and $x_2 > x_1$ at the instant shown in the figure, we can write the equations of motion using the balance of linear momentum as

$$\begin{aligned}
\text{Mass A:} \quad m\ddot{x}_1 &= k_2(x_2 - x_1) - 2k_1 x_1 \\
&= -(2k_1 + k_2)x_1 + k_2 x_2 \\
\text{Mass B:} \quad m\ddot{x}_2 &= -k_2(x_2 - x_1) - 2k_1 x_2 \\
&= k_2 x_1 - (2k_1 + k_2)x_2.
\end{aligned}$$

These two equations can be also written in a convenient matrix form as

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} = \frac{1}{m}\begin{bmatrix} -(2k_1 + k_2) & k_2 \\ k_2 & -(2k_1 + k_2) \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \tag{10.46}$$

$$m\ddot{x}_1 = -(2k_1 + k_2)x_1 + k_2 x_2, \quad m\ddot{x}_2 = k_2 x_1 - (2k_1 + k_2)x_2$$

2. The out of phase normal mode of vibration of the two masses is such that $x_1(t) = x_0 \sin \lambda_n t$ and $x_2 = -x_0 \sin \lambda_n t$, *i.e.*, the two masses have out of phase displacements ($x_1 = -x_2$). If we substitute these values of the displacements, we see that both equations turn out to be the same and they give,

$$-\lambda_n^2 x_0 \sin \lambda_n t = \frac{1}{m}(-2k_1 - k_2 - k_2)x_0 \sin \lambda_n t$$

from which it follows that,

$$\lambda_n = \sqrt{\frac{2(k_1 + k_2)}{m}}.$$

Substituting the given values of m, k_1, and k_2, we get,

$$\lambda_n = \sqrt{\frac{2(25+3)\,\text{N/m}}{34.5 \times 10^{-9}\,\text{kg}}} = 40.29 \times 10^3\ \text{rad/s}.$$

Thus the natural frequency corresponding to the out of phase vibration mode is 40.29×10^3 rad/s which corresponds to $f_n = \lambda_n/2\pi = 6.4$ kHz.

$$f_n = 6.4\text{kHz}$$